A MC approach to simulate up- and down-going neutrino showers including local topographic conditions

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Abstract

The extragalactic flux of protons is predicted to be suppressed above the famous Greisen–Zatsepin–Kuzmin cut-off at about $E_{\text{GZK}} \approx 50$ EeV due to the resonant photo-pion production with the cosmic microwave background. Current cosmic ray data do not give a conclusive confirmation of the GZK cut-off and the quest about the origin and the chemical composition of the highest energy cosmic rays is still open. Amongst other particles neutrinos are expected to add to the composition of the cosmic radiation at highest energies. We present an approach to simulate neutrino induced air showers by a full Monte Carlo simulation chain. Starting with neutrinos at the top of the atmosphere, the performed simulations take into account the details of the neutrino propagation inside the Earth and atmosphere as well as inside the surrounding mountains. The products of the interactions are input for air shower simulations. The mountains are modelled by means of a digital elevation map. To exemplify the potential and features of the developed tools we study the possibility to detect neutrino induced extensive air showers with the fluorescence detector set-up of the Pierre Auger Observatory. Both, down-going neutrinos and up-going neutrinos are simulated and their rates are determined. To evaluate the sensitivity, as a function of the incoming direction, the aperture, the acceptance and the total observable event rates are calculated for the Waxman–Bahcall (WB) bound.

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1. Introduction

The detection of very high energy cosmic neutrinos, above 1 EeV, is important as it may allow to identify the most powerful sources in the Universe.

In the first place, neutrinos, due to their small cross-section can travel cosmological distances without interactions. Hence they might carry astrophysical information about their sources which are commonly believed to be optically thick [1].

Second, due to their connection to the emission of cosmic nuclei and gamma rays, they might help to solve the problem of the origin of ultra high energy cosmic rays (UHECRs). In fact although the existence of UHECRs is experimentally proven, their composition and origin are still unknown. Many models have been proposed to explain the origin of UHECRs. Some of these, which involve mechanisms of particle acceleration (bottom-up), claim that they might be produced by Active Galactic Nuclei (AGN) and Gamma Ray Bursts (GRB) [2]. In such scenarios neutrinos could be produced with an upper flux limit given by the Waxman–Bahcall (WB) bound [1]. Other models claim that UHECRs might come from the decay of super-massive objects (top-down): these objects are expected to be produced by radiation, interaction or collapse of topological defects such as monopoles, cosmic strings, etc. [3]. The topological defect models predict a larger flux of photons and neutrinos arriving at Earth than the bottom-up models.

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Finally, high energy neutrinos could be also produced through pion decay when protons interact with the cosmic microwave background. These neutrinos are so-called cosmogenic neutrinos [4].

Nevertheless, due to their vacuum oscillations [5], a flux of high energy cosmic neutrinos is expected to be almost equally distributed among the three neutrino flavours. Therefore the study of oscillation effects on high energy neutrino fluxes can be used to study the neutrino mixing and distinguish amongst different mass schemes.

According to the models described above, a low incoming flux of neutrinos is expected. In addition, neutrinos have a small interaction probability. Therefore, in order to get a detectable rate, very large neutrino detectors are needed. Neutrinos can interact when passing through a large amount of matter such as the Earth or its atmosphere. Neutrinos can interact either through neutral current (NC) or through charged current (CC) deep inelastic scattering (DIS). In NC reactions, $\nu + N \rightarrow \nu + X$, a neutrino will scatter (inelastically) with a nucleon N in the same way for all types of neutrinos. In CC reactions, $\nu + N \rightarrow 1 + X$, a neutrino is converted into the corresponding charged lepton $l$, with different final states for the three flavours.

In the atmosphere, only very inclined electron neutrinos and tau neutrinos can initiate extensive air showers (EAS) with clear neutrino signatures detectable by ground detectors, see for example [6]. Muon neutrino induced EAS are generally weaker with a smaller energy transfer into the EAS, thus with a suppressed longitudinal profile and much less particles on ground. As a consequence the detection probability is reduced too [7].

Passing the Earth’s crust only tau neutrinos can finally produce detectable showers seen by a ground detector. Due to interaction the neutrino loses some of its energy in CC and NC scattering. For an initial $\nu_e$ an electron is created. The electron is rapidly brought to rest in matter. For an initial $\nu_\mu$ a muon is created. This lepton subsequently decays, producing a muon neutrino, an electron and an electron neutrino. However, before decaying the muon propagates through the matter, too. As its radiation range in matter is much smaller than its decay length, it loses most of its energy. The produced muon can occasionally escape from the Earth, but it does not decay in the atmosphere since the decay length is about $10^8$ times larger than the one of tau (see Section 2). For an initial $\nu_\tau$ a tau lepton is created. This lepton decays through one of the possible known decay modes, but a $\nu_\tau$ is always present as one of the decay products. Since the life time of a tau lepton is much smaller than the one of the muon, the Earth is not entirely opaque to tau neutrinos. Especially if the tau lepton is produced close to the Earth’s surface (Earth-skimming shower), it can emerge and decay in the atmosphere, producing a shower which is potentially observable by giant air shower detectors, such as the Pierre Auger Observatory [8]. These features tend to favour the idea that Earth-skimming $\nu_e$ showers are the best suited candidates to produce detectable showers in the Auger Observatory. Studies of such possibilities were recently presented in [6,9–12].

To exemplify the potential and the features of the developed simulation tool we study neutrino induced EAS focusing on the fluorescence detector (FD). The fluorescence detector measures the fluorescence light produced in the atmosphere by EAS. To set up the simulation chain for inclined neutrino induced showers, we have simulated in very detail neutrino interactions taking into account the local topographic conditions of the detector, the curvature of the Earth and the neutrino propagation inside the Earth and atmosphere.

Several recent papers address the simulation of neutrino induced EAS using a spherical model of the Earth not taking into account the influence of the local topographic conditions on calculated neutrino rate in detail. In this paper, we describe the procedure to simulate neutrino propagation more accurately. Taking into account the details of the neutrino propagation inside the Earth and in the atmosphere as well as the topography of the Pierre Auger Observatory site, the directional dependence of the neutrino rate for up-going and down-going neutrino showers (probability maps) are evaluated accounting for the 10% duty cycle for the FD. The Waxman and Bahcall (WB) bound is used as an initial neutrino flux. We use the ellipsoidal model of the Earth based on the so-called datum WGS84 [13] model. In addition, we use an elevation map of the mountains surrounding the Auger site. We find that the shape of the surrounding area influences significantly the calculated neutrino rates. To investigate the response of the FD for up-going showers, we generate the longitudinal profiles of shower development using the EAS MC generator AIRES [14]. The light propagation and the hardware detector trigger are simulated by means of the Auger software framework called Offline [15]. Finally the acceptance is calculated for up-going showers.

A recent semi-analytical approach covering this topic was published in [12]. In this paper we treat neutrino and tau lepton propagation in more detail. In particular, we obtain the decay vertex position, energy and momentum of decay products for simulated neutrino showers. Moreover, these quantities are simulated independent of the definition of the detector volume. It will be a useful tool for further calculations for other neutrino detectors.

The outline of the paper is the following. The method is described in Section 2. We report our results in particular on expected event rates for the fluorescence detector (FD) including the appropriate aperture and acceptance of the Pierre Auger Observatory in Section 3. Conclusions are given in Section 4.

2. Method

The assumption on the initial neutrino flux determines the number of neutrinos which have been simulated propagating through the Earth an atmosphere before they
possibly have initiated an EAS. The FD detector response of the resulting EAS is simulated too. The details of the method and tools applied are given below.

2.1. Neutrino flux

One of the possible source of UHECRs are GRBs. If this is the case, Fermi accelerated protons from shocks might generate extremely high energy neutrinos with an energy spectrum \( \Phi(E) \sim E^{-2} \). This neutrino flux, as well as those from other compact sources, such as AGN, is limited by the WB bound [1], see Fig. 1. This constraint is valid for all astrophysical neutrino sources that are optically thin to \( p\gamma \) and pp interactions. In this paper we use a rather conservative estimate of this bound

\[
E^2 \Phi(E) = 3 \times 10^{-8} \quad (\text{GeV s}^{-1} \text{cm}^{-2} \text{sr}^{-1}),
\]

where the flux is the sum of neutrinos and anti-neutrinos of all flavours.

While propagating to the Earth, the neutrino flavours are mixing [5]. Assuming mixing, the \( \nu_e + \bar{\nu}_e \) flux can given as \( \Phi(E_{\nu_e + \bar{\nu}_e}) = \frac{1}{2} \Phi(E_e) = 1 \times 10^{-8} E^{-2} \quad (\text{GeV s}^{-1} \text{cm}^{-2} \text{sr}) \).

2.2. Neutrino propagation

In the following we describe the code used to simulate neutrino propagations and interactions. The software tool used in the present paper is based on the code ANIS (All Neutrino Interaction Simulation) [16]. ANIS is a code originally developed by Gazizov and Kowalski for the complete simulation chain of neutrino propagation and interaction in the context of the AMANDA experiment [17]. It allows to generate \( \nu \)-events of all flavours, to propagate them through the Earth and finally to simulate \( \nu \)-interactions within a specified volume inside the Earth. All relevant Standard Model processes (charged current (CC), neutral current (NC) \( \nu N \)-interactions and resonant \( \bar{\nu}_e - e^- \) scattering) are implemented. In addition, neutrino regeneration at all orders is included. The density profile of the Earth is chosen according to the Preliminary Earth Model [18]. Deep inelastic \( \nu N \)-cross-sections are calculated from Quantum Chromo Dynamics (pQCD) with structure functions according to CTEQ5 [19] and with a logarithmic extension into the small-\( x \) region. The tau decay is simulated using the program TAUOLA [20].

In order to simulate neutrino showers suitable to be detected by the Pierre Auger Observatory, some important changes and extensions of the code were needed. For instance, in ANIS neutrino propagation and tau decay are simulated only inside the Earth. Instead, we need to simulate also the propagation through the atmosphere. Moreover, it is necessary to take into account that the Auger Observatory is positioned on the surface of the Earth.

Below we present the main extensions of the ANIS code. First, the topography of the Auger site was implemented. The description of the relief of the Andes mountains was made according to a digital elevation map (DEM). These data are available from the Consortium for Spatial Information (CGIAR-CSI) [21]. The data distributed by CGIAR-CSI are in ArcInfo ASCII and GeoTiff format, in decimal degrees and datum WGS84 [13]. They have a resolution of 90 m at the equator and are provided in mosaiced 5° × 5° tiles for easy download. For our purpose the resolution is too high and therefore a reduction of data points was made to save memory space and speed up the simulations. The data points were transformed to the global ANIS Cartesian coordinate system. The map of the area around the Auger site obtained is shown in Fig. 2. A final map with a fixed cell size of 5 km × 5 km was created. To facilitate a continuous transition of the Earth’s surface between the DEM profile and the spherical Earth model used in ANIS, the Earth’s radius was reduced from

![Fig. 1. Total neutrino fluxes as a function of energy in case of the WB bound [1]. The lower thin solid horizontal line corresponds to WB bound assuming proton–proton interaction; the upper thin solid line gives the WB upper bound corrected for neutrino energy loss due to redshift (see [1] for details). The thick solid line is the flux based on the WB bound used in this paper. The dashed lines stand for an initial neutrino flux used in Ref. [9,12] for the WB bound. In all cases, the curves show the sum of neutrinos and anti-neutrinos for all flavours.](image1)

![Fig. 2. Topography of the Auger site according to CGIAR-CSI data [21]. The centre of the map corresponds to the centre of the Auger array (latitude \( \phi_{\text{Auger\_center}} = 35.25^\circ \) S, longitude \( \lambda_{\text{Auger\_center}} = 69.25^\circ \) W). The Auger position is marked by a circle.](image2)
6378 km (value used by ANIS) to 6371 km corresponding to the radius at the centre of the Auger site.

Second, a redefinition of the detection volume was done. The original ANIS version uses the concept of the detection volume \[\text{det vol} \] [22]. This concept is useful for calculating event rates for a given neutrino flux \(\Phi(E, \theta)\). The detection volume corresponds to the so called \textit{active volume} in which potentially detectable neutrino interactions are simulated. In the original ANIS this volume is defined as a cylinder with \(z\)-axis parallel to the neutrino direction [16]. In our adapted version we kept the general idea of \textit{active volume}, but with some modifications, as shown in Fig. 3. As it can be seen from this figure, the \textit{active volume} for a given incoming neutrino with energy \(E\), is defined by a particular plane \(A_{\text{gen}}\) and distance \(\Delta L\). The plane \(A_{\text{gen}}\) is the cross-sectional area of the detector volume and it was used as reference plane for the generation of incoming neutrinos. The area depends on the zenith angle \(\theta\) of the incoming neutrino. The detector was modelled as a cylinder with radius \(R\) and height \(H\), which is given by

\[
A_{\text{gen}}(\theta) = \pi R^2 \cos \alpha + 2HR \sin \alpha, \tag{2}
\]

where \(\alpha = \begin{cases} \theta & \text{for } 0 \leq \theta < \pi/2, \\ \pi - \theta & \text{for } \pi/2 \leq \theta < \pi. \end{cases} \)

The distance \(\Delta L\) is the multiple, \(n\), of the average lepton range \(\langle R_{\text{lep}}(E_{\text{lep}}) \rangle\) [23]. Inside the \textit{active volume} lepton tracks are shown also. Neutrinos with energy \(E\), produce leptons with individual ranges, depending on the fraction of energy transferred. More precisely, the incoming neutrino is forced to interact in the \textit{active volume} according to its interaction probability

\[
P(E, E_{\text{lep}}, \theta) \simeq N_A \times \sigma(E) \times \rho(Z) \times \Delta L, \tag{3}
\]

where \(\sigma(E)\) is the total neutrino cross-section, \(\rho(Z)\) the local medium density and \(N_A\) the Avogadro constant. \(P(E, E_{\text{lep}}, \theta)\) is the probability that a neutrino with energy \(E\), crossing the distance \(\Delta L\) would produce a lepton with an energy \(E_{\text{lep}}\). In this way the production vertex of the lepton is created. Then, the lepton propagates through the matter, loses some energy and decays (if it is unstable) in the vicinity of the detector or inside the detector volume. Here, two important comments are in turn. First, due to the energy dependence of the height of the \textit{active volume}, the volume could be sometimes smaller than detector volume. In such cases the detector volume was used as the \textit{active volume}. Second, during the propagation of the lepton inside the \textit{active volume} the lepton may cross the Earth’s surface. In such cases different energy losses in Earth and air are taken into account.

In order to calculate physical quantities, one has to weight the events. A first weight is the interaction probability defined by Eq. (3). A second weight comes from the normalisation of the injected neutrino flux \(\Phi(E, \theta)\)

\[
F^w(E) = \frac{N_{\text{gen}}^{-1}}{\int_{E_{\text{min}}}^{E_{\text{max}}} \Phi(E) \, dE} \times \int_{\cos \theta_{\text{min}}}^{\cos \theta_{\text{max}}} A_{\text{gen}}(\theta) \times d\Omega, \tag{4}
\]

where \(d\Omega = 2\pi \sin \theta \, d\theta\) is the space angle, \(\Delta T\) the observation time and \(N_{\text{gen}}\) is the number of generated events from surface \(A_{\text{gen}}\). Here, we assume the isotropic neutrino flux \(\Phi(E, \theta)\). The expected event rate of the neutrinos in the detector volume can be calculated from

\[
N_{\text{ratio}} = F^w(E) \times \sum_{i=1}^{N_{\text{acc}}} P_i, \tag{5}
\]

where \(N_{\text{acc}}\) is the number of events triggering the detector and passing all quality cuts of the cascade analysis. The normalisation factor \(F^w(E)\) is chosen such that the rate of the neutrinos results in the total number of events per year.

Third, new tau lepton decay channels were implemented. In the original ANIS code the tau lepton decay is simulated using the code of TAUOLA [20]. The tau lepton decay is simulated by randomly choosing one event from a data base of pre-simulated events. The data base is a table with 10000 final states. The table contains the fractional energy of the hadronic and electromagnetic particles, muons and all types of neutrino flavours. Cherenkov neutrino detectors cannot see the difference between an electron and a photon. Both lead to an electromagnetic (EM) shower of equal energy and the difference of interaction lengths is not resolvable. Hence the original ANIS code writes out every secondary particle from the tau lepton decay, which is able to produce a purely EM shower, as a single electron. A similar procedure is done for hadrons. ANIS dumps all hadronic energy into a single pion of corresponding energy. For underground neutrino detectors such as IceCube [24] or AMANDA this approach...
works quite well because the final detectable states are much less complex and it allows to simplify the code. For detectors positioned in air such as the Pierre Auger Observatory this simplification does not work. For example, the response of an initial $\pi^-$, produced in the decay of a tau lepton, is different from that of a $2\pi^-$, see Table 1. In our version of ANIS the table of the final states has been extended, based on TAUOLA simulations. Sixty thousand TAUOLA events are simulated and 10000 events were chosen according to the decay channels and probabilities (branching ratios) listed in Table 1. Finally, the table of the final states was randomised to get an unsorted list. Since TAUOLA gives a 4 momentum in the central mass (CM) system of all decay particles, a Lorentz transformation of the energy is done to obtain the energy fraction in the laboratory (LAB) system. In Fig. 4, the fraction of energy in the LAB frame for different particles was shown for one tau lepton decay channel. We have seen an excellent agreement between old and new distributions.

Finally, the tau lepton decay routine was partly rewritten to take into account the processes of tauons escaping from the Earth’s surface and decaying in the atmosphere. In ANIS, the tau is propagated in small energy steps until the age of the tau lepton exceeds the tau lepton lifetime. This procedure works quite well if the production and decay vertex are in the same medium (either in air or rock), but if the tau lepton crosses the Earth’s surfaces during its propagation inside the active volume, the different amount of energy loss in the Earth’s crust and air have to be taken into account. In particular, when a tau lepton is generated in the Earth, it loses energy due to ionisation and radiation processes. These energy losses per unit length of crossed matter (in g/cm$^2$) is usually approximated by a linear equation (continuous energy loss approach), which reads as

$$\frac{dE}{dX} = -\alpha \beta(E) \times E,$$

where the factor $\alpha$ is due to ionisation losses and $\beta$ is due to radiation losses. The factor $\alpha$ is negligible for ultra-high energies. The factor $\beta$ parameterises the tau lepton energy loss through bremsstrahlung, pair production, and photo-nuclear interaction. Exemplary we have used two parameterisations: $\beta$ is linear dependent on energy.

Table 1
The decay channels implemented in the modified ANIS software

<table>
<thead>
<tr>
<th>Decay</th>
<th>Secondaries</th>
<th>Probability</th>
<th>Air-shower</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau \rightarrow \mu^- \nu_\mu \nu_\tau$</td>
<td>$\mu^-$</td>
<td>17.4%</td>
<td>Unobservable</td>
</tr>
<tr>
<td>$\tau \rightarrow e^- \nu_e \bar{\nu}_\tau$</td>
<td>$e^-$</td>
<td>17.8%</td>
<td>1 Electromagnetic</td>
</tr>
<tr>
<td>$\tau \rightarrow \pi^- \nu_\tau$</td>
<td>$\pi^-$</td>
<td>11.8%</td>
<td>1 Hadronic</td>
</tr>
<tr>
<td>$\tau \rightarrow \pi^- \pi^0 \nu_\tau$</td>
<td>$\pi^-, \pi^0 \rightarrow 2\gamma$</td>
<td>25.8%</td>
<td>1 Hadronic, 2 electromagnetic</td>
</tr>
<tr>
<td>$\tau \rightarrow \pi^- 2\pi^0 \nu_\tau$</td>
<td>$\pi^-, 2\pi^0 \rightarrow 4\gamma$</td>
<td>10.79%</td>
<td>1 Hadronic, 4 electromagnetic</td>
</tr>
<tr>
<td>$\tau \rightarrow \pi^- 3\pi^0 \nu_\tau$</td>
<td>$\pi^-, 3\pi^0 \rightarrow 6\gamma$</td>
<td>1.23%</td>
<td>1 Hadronic, 6 electromagnetic</td>
</tr>
<tr>
<td>$\tau \rightarrow \pi^- \pi^- \pi^- \nu_\tau$</td>
<td>$2\pi^-, \pi^-$</td>
<td>10%</td>
<td>3 Hadronic</td>
</tr>
<tr>
<td>$\tau \rightarrow \pi^- \pi^- \pi^- \pi^- \nu_\tau$</td>
<td>$2\pi^-, \pi^-, \pi^0 \rightarrow 2\gamma$</td>
<td>5.18%</td>
<td>3 Hadronic, 2 electromagnetic</td>
</tr>
</tbody>
</table>

The table was taken from [33].

Fig. 4. Fraction of energy in the LAB frame for one tau decay channel: $\tau \rightarrow e^- \nu_e \bar{\nu}_\tau$. The dashed line corresponds to the fraction of energy of the tau final states obtained from the table used in the original ANIS version. The solid line corresponds to the new distribution generated using TAUOLA.
The tau lepton energy loss in standard rock according to Eq. (6) is shown for the parameter set to the Preliminary Earth Model [18] and for different parameterisation of the \( q \).

\[
\begin{align*}
\beta_A(E_t) & = 0.71 \times 10^{-6} \text{ cm}^2 \text{ g}^{-1} \\
& + 0.35 \times 10^{-18} E \text{ cm}^2 \text{ g}^{-1} \text{ GeV}^{-1}
\end{align*}
\]

(7)

used by Aramo et al. in Ref. [9], and

\[
\beta_B(E_t) = (1.508 + 6.3(E_t/10^{18})^2) \times 10^{-7} \text{ cm}^2 \text{ g}^{-1}
\]

(8)

from [16, 23, 25].

The different parameterisations lead to rather important changes of the tau lepton energy loss per unit length of crossed matter, as shown in Fig. 5. A difference of about 30% between parameterisation \( \beta_A \) and \( \beta_B \) is seen for an energy of around 10 EeV. The lepton range, \( R_\tau \), is given by the integral of the inverse of the tau lepton loss rate over the tau lepton energy

\[
R_\tau = \frac{1}{\rho(z)} \int \frac{1}{dE/dX} \, dE.
\]

(9)

Thus differences in \( \beta \) will lead to different tau lepton ranges (length of the lepton track) and consequently influence the expected neutrino event rate and aperture calculations.

The tau lepton is propagated in small steps of energy until its age in the tau rest-frame exceeds its lifetime. During a single step the distance passed by the tau lepton is evaluated according to the following formula:

\[
\Delta R_\tau = \frac{1}{\rho(z)\beta(E_t)} \ln(E_t/E_i),
\]

(10)

where \( E_i \) is the initial energy of the tau lepton and \( E_f \) is the energy of the tau lepton after the distance \( \Delta R_\tau \). During the energy step \( \beta \) is constant. The distance \( \Delta R_\tau \) is evaluated with the altitude dependent density \( \rho(z) = \rho_{\text{rock}}(z) \) or \( \rho(z) = \rho_{\text{air}}(z) \). \( \rho_{\text{rock}} \) is the density of the Earth according to the Preliminary Earth Model [18] and \( \rho_{\text{air}} \) is the air density calculated according to the US standard atmosphere (Linsley parameterisation) [26].

In Fig. 6, the tau lepton range in standard rock, \( \rho = 2.6 \text{ g/cm}^3 \) is shown for the parameter set \( \beta_B \) based on calculations reported in [25]. One can see that the tau lepton range \( R_\tau \) is of the order of about 10 km at 1 EeV. This value is about 5 times smaller than the tau lepton decay length, \( d_\tau \), at the same energy. The decay length is given by \( d_\tau = c\tau(E_t/m_\tau) \sim 49 \text{ km} \times (E_t/10^{18} \text{ eV}) \) for tau lepton, where \( \tau_t = 87.11 \) is the tau lepton lifetime in \( \mu \text{m} \) and \( m_\tau = 1777.03 \text{ MeV} \) is the tau lepton mass [27]. Thus, a tau lepton with an energy of about 1 EeV produced close to the Earth’s surface can escape from the Earth before decaying. One can note also, that the tau lepton decay length is about 490 km for a tau lepton energy of about 10 EeV. Therefore an emerging tau lepton can escape from the atmosphere, which is assumed to have an height of 100 km.

2.3. Detector properties and neutrino induced shower simulation

The Pierre Auger Observatory is being built as a hybrid-detector for the detection of EAS as it combines the measurement of the particle density on ground – by means of a huge array of water Cherenkov detectors – with the measurement of the fluorescence light produced by EAS in the atmosphere. The completed surface detector array will consist of about 1600 water Cherenkov detectors (tanks) arranged in a triangular grid with 1.5 km spacing. The fluorescence detector (FD) will consist of 4 telescope buildings (eyes) overlooking the detector array. Each building houses 6 telescopes with a 30° x 28.6° viewing angle, leading to a coverage of 180° in azimuth for each eye. The fluorescence light is focused in each telescope onto a camera consisting of 440 photo-multipliers (PMTs) through its Schmidt-optics and a spherical mirror of 17 m². The FD detector probes the longitudinal development of EAS by measuring the photon emission from atmospheric nitrogen, which is excited by the charged particles of the shower.

On top of the expected event rates calculated inside a defined volume below, we would like to exemplified determine the acceptance of FD for the Pierre Auger Observatory in the case of up-going showers. Applying the neutrino generation code we obtain the tau lepton decay vertex position, energy and momentum of the decay products for simulated neutrino showers with given energy. AIRES was used to generate the longitudinal profiles of charged particles and the energy deposit based on the
ANIS output. A special mode was used to inject simultaneously several particles at a given interaction point. The emission of fluorescence light by the shower together with its propagation towards the detector and the response of detector itself, including electronics and trigger was simulated by the Offline software, already mentioned before. Finally on the basis of the algorithms implemented in the Offline software the first two trigger levels called First Level Trigger (FLT) and Second Level Trigger (SLT) are simulated [8]. After the simulated events had passed the FLT threshold trigger (at least one pixel), a search for pattern consisting of 5 pixels was performed according to the SLT algorithm. In this way the trigger efficiency of the FD for a given neutrino energy can be defined as

\[ \Sigma(E_\nu) = \frac{N_{\text{SLT}}}{N_{\text{AIRES}}} \times \gamma \]

where \( N_{\text{AIRES}} \) is the number of AIRES tau showers simulated for a given neutrino energy, \( N_{\text{SLT}} \) the number of showers passing the SLT condition and \( \gamma \) the duty cycle of fluorescence detector.

### 3. Results

Simulations are done for neutrino showers ranging from 1 to 100 EeV. Usually 500,000 events are generated on the surface \( A_{\text{gen}} \) for different azimuth and zenith angles. The incoming neutrino is forced to interact at a distance \( \Delta L \), which corresponds to an average of about 15 lepton ranges. According to Fig. 6, \( \Delta L \approx 150 \text{ km} \) for an initial neutrino energy of 1 EeV and \( \Delta L \approx 300 \text{ km} \) for an energy of 100 EeV. In this way the lepton production vertex is generated in almost the whole area presented in Fig. 2.

A cuboid with an area of \( 50 \times 60 \text{ km}^2 \) and height of 10 km positioned at 1430 m a.s.l. was used as detector volume (\( \equiv V_{\text{FD}} \)). It agrees quite well with the detection volume seen by the Auger fluorescence detectors (FD). However, we must stress that other geometries can be easily adopted. Here, the expected number of events (event rate) was calculated for an energy threshold of \( E_{\text{th}} = 1 \text{ EeV} \), assuming a 10\% duty cycle of the FD detector (optical observations are limited to moonless and clear nights only).

To test our simulation chain, and in particular the production of tau leptons, the distribution of the inelasticity parameter for CC \( \nu_e \) reactions is calculated. In this process the energy of tau neutrinos is transferred to the struck quark of the target nucleon causing the nucleon to break. The fraction of neutrino energy that is transferred to the quark is denoted by \( y \). For the CC reaction \( y \approx 1 - E_\nu/E_\nu^c \), where \( E_\nu^c \) is the energy of the incident neutrino and \( E_\nu \) is the energy of the tau lepton produced in the reaction. For ultra high energy neutrinos the mean inelasticity parameter is \( \langle y \rangle \approx 0.2 \) [10]. The inelasticity parameter is shown in Fig. 7 from our MC simulation. One can see that the average value \( y = 0.22 \) agrees very well with the canonical value 0.2.

### 3.1. Downward-going tau neutrinos showers

In this section the influence of the atmosphere and different models of energy losses on the event rate is given. We also present the probability map (event rate) evaluated taking into account the local topographic conditions of the Auger site.

We study the expected event ratio for \( \nu_e \) and \( \nu_\mu \) downward-going (DW) showers calculated on the basis of the spherical model of the Earth and the WB neutrino flux discussed above. Our results are presented in Table 2. One can see that the expected event rate for \( \nu_e \) and \( \nu_\mu \) agrees within 33\% (42\%) with corresponding results from Ref. [12] (the last column of Table 2).

For \( \nu_e \) the main difference comes from the use of different models for the cross-section and models for the atmosphere. In this paper the parameterisation of cross-section based on CTEQ5 parton distribution function was used and the air density was calculated according to the US standard atmosphere. Since there is no additional effect which can influence the calculated rate, such as for example the lepton energy loss, the calculated rate should depend only on the cross-section and air density according to Eqs. (3) and (5). Both quantities can contribute to the difference observed between our rate and the rate calculated by [12] for DW showers. During propagation of \( \nu_e \) through

![Fig. 7. Distribution of the inelasticity parameter for CC \( \nu_e \) interactions. The mean inelasticity parameter \( \langle y \rangle \) is 0.22.](image)

<table>
<thead>
<tr>
<th>Flavour</th>
<th>( \lambda_{\text{FD}} ) (yr(^{-1}))</th>
<th>( N_{\text{acc}} )</th>
<th>( \lambda_{\text{FD}} ) (yr(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \nu_e )</td>
<td>0.019 ± 2%</td>
<td>121045</td>
<td>0.034</td>
</tr>
<tr>
<td>( \nu_\mu )</td>
<td>0.006 ± 2%</td>
<td>40190</td>
<td>0.009</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>0.025 ± 3%</strong></td>
<td></td>
<td><strong>0.043</strong></td>
</tr>
</tbody>
</table>

The uncertainties of the event rate are calculated according to a bootstrap method. For \( \nu_e \), the rate is calculated according to the model of energy loss \( \beta_A \) given in Ref. [9].
the atmosphere and inside $V_{FD}$ we need to consider the fact that the density of the air is not constant. This effect will lead to a smaller interaction probability and consequently to a smaller value of the event rate. To check how changes of the air density influence the calculation of the event rate we repeat the calculation with a constant air density corresponding to the value at sea level, $\rho = 0.00122 \text{ g/cm}^3$ for all altitudes. As a result the rate is about 63% larger (i.e. $N_{\text{FD}} = 0.031 \text{ yr}^{-1}$) than rate presented in Table 2 for $v_e$ and for a spherical model of the Earth. This example demonstrates the importance of considering the atmosphere for the event rate calculation of DW showers.

The rates presented in this paper are affected by uncertainties of the $v$-nucleon cross-section. Here the CTEQ5 parameterisation of the cross-section was used instead of CTEQ6 as used in Ref. [12]. The CTEQ5 parameterisation is almost the same like CTEQ4 [16,28] in contrast to CTEQ6 which predicts a slightly larger cross-section than CTEQ4 (about 15% in energy range from 1 to 100 EeV, see Fig. 5 in Ref. [9]). Thus we should expected a slightly larger rate than the rate calculated in [12], but we have to remember that due to the atmosphere effect discussed above the expected rate could be smaller than rate calculated in Ref. [12] for DW showers.

For $v_e$, an additional difference comes from a different treatment of the tau lepton energy loss in air. In Ref. [12] the energy loss of leptons in air is not taken into account, while in this paper it is calculated according to Eq. (10) with different parameterisations of $\beta$. It is well known that larger values of $\beta$ lead to smaller values of the calculated event rate, see for example [6] and discussions in the next sections.

For quasi-horizontal $v_e$ showers (the zenith angle $\theta$ in the range $85^\circ-90^\circ$) we take into account the local topographic conditions of the Auger site (see Fig. 2). In this case, neutrinos produce a tau lepton in the atmosphere or in the mountains surrounding the Auger detector. In Fig. 8, the corresponding tau lepton spectrum is shown. The power law dependence due to the used WB spectrum is seen for higher tau lepton energies ($E_\tau > 1 \text{ EeV}$). There are also tau leptons with energies smaller than 1 EeV, which are produced by low energetic neutrinos. The initial neutrino interacts with the mountains surrounding the Auger site and produces a tau lepton. The tau lepton, due to its short lifetime (the tau lepton range for this energy is below 10 km), rapidly decays into a $v_e$ with lower energy. This secondary neutrino interacts within the detector volume, $V_{FD}$ and again produces a tau lepton with lower energy. This neutrino regeneration effect leads to the significant reduction of the neutrino and tau lepton energy and it plays a more important role for up-going neutrino showers.

In Fig. 9, the map of the expected event rate in $\text{yr}^{-1}$ as a function of the incoming direction ($\theta, \phi$) is shown. Significant directional differences in the number of expected events are seen. The largest rate is for almost horizontal neutrinos with $\theta$ in the range between $89^\circ-90^\circ$ and the azimuth $\phi$ in the range between $100^\circ$ and $270^\circ$ (North–West–South). However, some peaks in the rate distribution exist for an azimuth of about $350^\circ$ (East). This behaviour is due to the fact that the Auger site is surrounded by Andes on the West and smaller but closer mountains in the East (see the map shown in Fig. 2).

In Table 3, the expected event rate is listed for predefined azimuth and zenith angle ranges, in case of quasi-horizontal neutrino showers. The ratio is calculated for tau leptons with an initial energy larger than the threshold

![Fig. 9. Event rate as a function of azimuth and zenith for the active volume in the case of quasi-horizontal $v_e$ showers, assuming $E_\nu^0 = 0 \text{ EeV}$, a WB flux, $\Delta L = 15 (R_s/E_s)$, $\beta_B$, $N_{\text{FD}} = 0.071 \text{ yr}^{-1}$.](image)
energy \((E_{th} = 1\text{ EeV})\) and a decay vertex position inside \(V_{FD}\). As in Fig. 9, the dependence of the expected number of events from the incoming neutrino direction is well seen. For example, in the case of quasi-horizontal showers the ratio from the West is about 5 times larger than from the East. This is mainly due to the largest amount of rock encountered by incoming neutrinos from the West. Additionally in the same table the values of the event rates calculated according to the model of energy loss with \(\beta_A\) from Ref. [9] are listed. There is a difference of about 20% on average in the calculated total rate between the parameterization discussed above and the \(\beta_B\) parameterisation.

3.2. Up-going and Earth-skimming \(\nu_e\) showers

The probability maps and the energy spectrum are presented in case of Earth-skimming \(\nu_e\) showers. In addition the influence of the mountains on the calculated event rate is estimated and compared with the latest results presented in Ref. [12]. Finally the aperture and acceptance for Pierre Auger Observatory is given.

In the case of up-going \(\nu_e\) the regeneration effect was taken into account in the present paper. A high-energy \(\nu_e\) interacts in the Earth producing a tau lepton which in turn decays into a \(\tau\) with lower energy due to its short lifetime. The regeneration chain \(\nu_e \rightarrow \tau \rightarrow \nu_e \rightarrow \ldots\) continues until the tau lepton reaches the detector (in our case the active volume). This effect leads to a significant enhancement of the tau lepton flux up to about 40% more than the initial cosmic flux of tau neutrinos of energies between \(10^6\) and \(10^8\) GeV [29,30].

In Fig. 10, the spectrum of the tau leptons is shown. The influence of the neutrino regeneration is clearly seen. There is a peak visible at about \(10^6\) GeV. The position of this peak depends on the incoming zenith angle, i.e. order of regeneration, and thus shifts to higher energies for smaller zenith angles.

In Fig. 11 the map of the expected event rate in yr\(^{-1}\) is presented for ES showers with \(\theta\) in the range between \(85^\circ\) and \(95^\circ\). The map is given for all tau leptons created in the active volume. To be more precise, Earth-skimming neutrino showers are such showers where the neutrino propagates through rock, irrespective whether it emerges from the spherical Earth or the surrounding mountains modelled by the DEM. To take this effect into account, we refer to Earth-skimming (ES) showers as showers belonging to class A or B (defined in Fig. 3) with an incoming direction within \(5^\circ\) above and below the horizon.

In Fig. 12, the rate distribution for tau leptons with an initial energy larger than the threshold energy \((E_{th} = 1\text{ EeV})\) and a decay vertex position inside \(V_{FD}\) is shown. We conclude that the correlations between the calculated distribution of the event rate and the topography of the Auger site is well seen (the mountain effect). Only quasi-horizontal neutrinos with an incoming zenith angle of about \(1^\circ\) above the horizon and about \(3^\circ\) below horizon can produce a significant contribution to the total event rate. Moreover, we can see some kind of background which comes from low energetic tau leptons (from neutrino regeneration). This background is reduced by an energy and geometrical cut of the detected leptons (compare Figs. 11 and 12 for \(\theta > 93^\circ\)). These general conclusions agree quite well with other results [9,12]. The mountain effect is much more pro-

![Fig. 10. Energy spectrum of tau leptons inside the active volume.](image)

![Fig. 11. Event rate as a function of azimuth and zenith for the active volume, in the case of Earth-skimming \(\nu_e\) showers \((85^\circ < \theta < 95^\circ)\), assuming \(E_{th} = 0\text{ EeV}, \text{a WB flux, } \Delta E = 15(R_i(E_i), \beta_B, N_{total}^{FD} = 4.07 \text{ yr}^{-1}\).](image)

![Fig. 12. Corresponding event rate as a function of azimuth and zenith for the detector volume \(V_{FD}\) and tau lepton energy \(E_i > 1\text{ EeV}, \beta_B, N_{total}^{FD} = 0.052 \text{ yr}^{-1}\).](image)
nounced for $\nu_\tau$ showers very close to the horizon and less obvious for larger zenith angle ranges. A quantitative analysis of this effect for ES is presented in Table 3 and for up-going showers in Table 4.

In Table 3, the dependence of the expected number of events from the direction of incoming neutrinos is well seen. For example, in the case of ES showers the expected ratio from the West is about 3 times larger than the ratio from the East (for quasi-horizontal shower the ratio from the West is about 5 times larger than from the East). Note also that the rate in the case of ES showers is about one order larger than the rate for quasi-horizontal showers.

In Table 4, the expected event rates for different ranges of zenith angle are listed. The rate is dominated by events close to the horizon and gets only small contributions at larger zenith angles. Additionally, the rate for the different classes of events defined in Fig. 3 is presented. Class A has the production vertex in the Earth (below horizon) and decay vertices inside $V_{FD}$. Class B and C have production vertices above the horizon and decay vertices inside $V_{FD}$ (the contribution of class C is about one order smaller than class B). In principle the difference between these classes are a measure of the influence of the mountain effect reflected in a zenith dependent event rate [31]. As it can be seen from this table (the last column) the contribution of mountains to the total $\nu_\tau$ event rate is significant. For example, for nearly horizontal $\nu_\tau$ showers within zenith angle less than 2° below the horizon, the contribution is about 50% while for larger zenith angle ranges it is on average less than 24%.

We come to the same conclusion, if we estimate the mountain effect taking into account the calculation with the simple spherical model of the Earth because the rate calculated in this case agrees very well with the rate calculated for class A. Note also that our rate, $N_{SP}^{FD} = 0.032 \text{ yr}^{-1}$ calculated for a zenith angle between 90° and 98° for class A and with the spherical model of the Earth (multiplied by a factor 10 to get 100% detector duty cycle and by a factor 1.33(3) to get the same WB flux) agrees rather well with the recent results presented in Ref. [31] for Earth-skimming (ES) showers (see Table 1 in this paper).

In Table 5, our results and the corresponding results from Ref. [12] are listed. A direct comparison of the two calculations is difficult because the incoming tau neutrino flux used here is different than the one given in Ref. [12], as it is shown in Fig. 1. Also the treatment of tau lepton propagation in air is different. In Ref [12], the energy loss of tau leptons in air is not taken into account, while in this paper it is calculated according to Eq. (9). However, it is worth mentioning that the total rate calculated for up-going showers with $\theta < 95°$ agrees within 9% with the rate calculated by [12] assuming that the main contribution to the total rate calculated by [12] comes from almost horizontal showers. Larger differences within ~27% in the rate exist for different directions, apart from the rate calculated for the West. In this case our result is the same than the one calculated by [12]. As it is shown in Table 3 the different models of the tau lepton energy loss (different factor $\beta$) can lead to significant differences in the expected event ratio (about ~20% for quasi-horizontal and ~10% for ES showers). Note that the event ratio becomes larger for smaller values $\beta$. In principle we should expect therefore a larger event rate calculated by [12] than our rate if we assume comparable detector geometries and neutrino fluxes.

In Fig. 13A, the estimation of the aperture for up-going $\nu_\tau$ showers is shown. We define the aperture by

$$A(E_\nu)|_{E_\nu>1}\text{EeV} = N_{\text{gen}}^{-1} \times \sum_{j=1}^{N_{\text{acc}}} \sum_{\phi=1}^{N_{\phi}} P_{ij}(E_\nu, E_\tau, \theta) \times A_j(\theta) \times \Delta \Omega,$$

(12)

where $N_{\nu}$ is the number of tau leptons with energy $E_\tau$, $E_\nu$ is the initial neutrino energy, $N_{\text{acc}}^{\nu,\phi}$ is the number of tau leptons coming from a given direction inside $V_{FD}$ and passing our cuts. The aperture was calculated according to the parameterisation of $\beta_A$ and $\beta_B$. Different parameterisations
tance from our MC calculation for one fluorescence eye of Pierre Auger Observatory, called Los Leones, is shown.

It is worth to mention that we obtain almost the same results displayed in Fig. 13B if we use the estimation of the FD efficiency given in Ref. [32]

\[
\Sigma(E, r) = \gamma \times \Sigma'(E) \times \Sigma''(r),
\]

where \(\Sigma'(E)\) is the efficiency depending on the energy and \(\Sigma''(r)\) is the efficiency depending on the distance of the EAS to the eye by a Gaussian distribution.

To calculate the aperture, the same \(\Sigma'(E)\) as the one in Ref. [32] was used but \(\Sigma''(r)\) was approximated by a Gaussian distribution depending on the distance to the shower maximum instead of the shower core. A Gaussian distribution centred at a distance of 15 km from the eye and a spread of 5 km was used. The Gaussian distribution of the core position in Ref. [32] was replaced by the distribution to the shower maximum, since the core is not well defined for very inclined showers. We consider only simulated events for which the maximum of the shower development was in the field of view of fluorescence detector. We can see that the acceptance calculated in this way agrees quite well with the MC result obtained for the detector station Los Leones.

4. Summary

A tool to simulate neutrino propagation and interaction in the Earth and in the atmosphere taking into account the local topographic conditions was presented. A detailed investigation of neutrino induced showers was performed. The focus was put on the neutrino sensitivity for the FD detector of the Pierre Auger Observatory and therefore we have used the digital elevation map for this site. The probability map (event rate distribution) and the aperture was calculated and the impact of the mountains surrounding the Auger site was quantified. General features are consistent with previous results, but as a consequence of the surrounding mountains a pronounced variation of the event rate as a function of the azimuth is seen.

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References


