Method to calculate the β exponent of the Heitler–Matthews model of hadronic air showers

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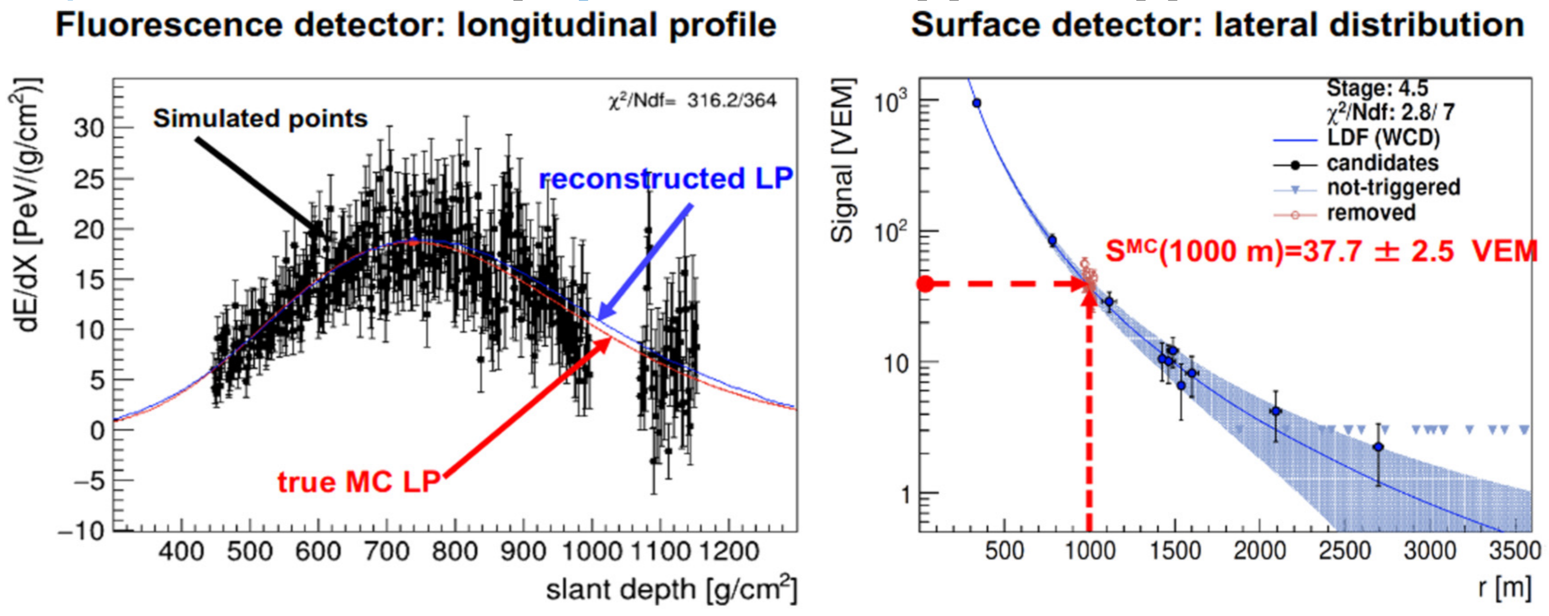
The number of muons in an air shower is a strong indicator of the mass of the primary particle and increases with a small power of the cosmic ray mass by the β-exponent, $N_\mu \sim A^{(1-\beta)}$. This behaviour can be explained in terms of the Heitler-Matthews model of hadronic air showers. In this paper, we present a method for calculating β from the Heitler-Matthews model. The method has been successfully verified with a series of simulated events observed by the Pierre Auger Observatory at 10^{19} eV. To follow real measurements of the mass composition at this energy, the generated sample consists of a certain fraction of events produced with p, He, N and Fe primary energies. Since hadronic interactions at the highest energies can differ from those observed at energies reached by terrestrial accelerators, we generate a mock data set with β = 0.92 (the canonical value) and β = 0.96 (a more exotic scenario). The method can be applied to measured events to determine the muon signal for each primary particle as well as the muon scaling factor and the β-exponent. Determining the β-exponent can effectively constrain the parameters that govern hadronic interactions and help solve the so-called muon problem: hadronic interaction models predict too few muons relative to observed events. In this paper, we lay the foundation for the future analysis of measured data from the Pierre Auger Observatory with a simulation study.

1) Introduction

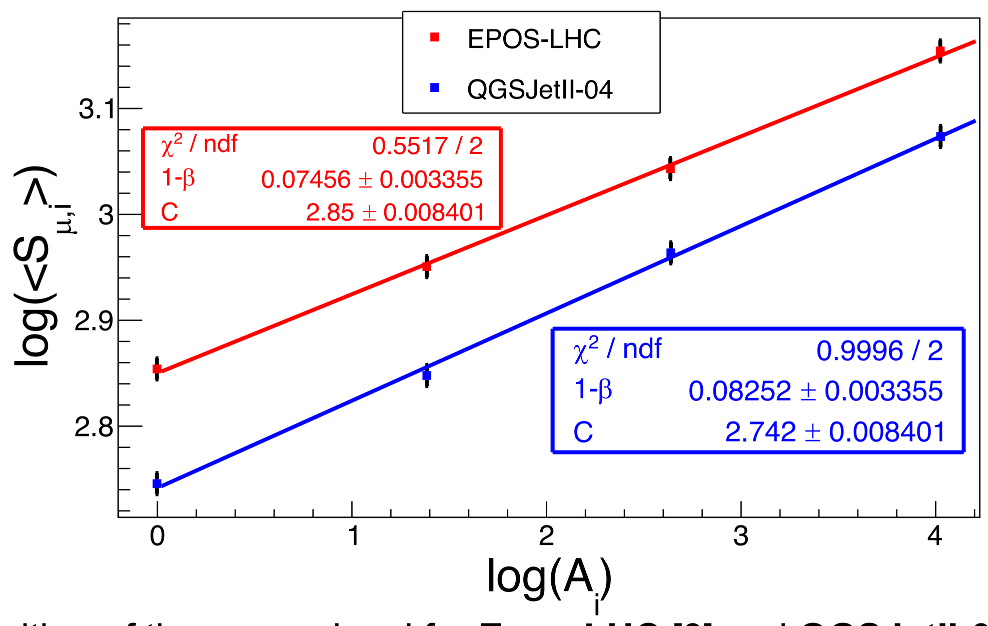
❖ **Top Down simulation chain[1-4]: Conex/CORSIKA[6]/OFFLINE[7]**



For each observed hybrid shower, starting with a large number of simulated air showers with varying initial conditions, we select the one which has a longitudinal profile most similar to the profile of the observed shower (the reference profile). As a result of the simulation-reconstruction chain we get an event, with complete information about the distributions of the signals in the detectors (including information on the specific components that contribute to these signals) – these signals can then be compared with their reference counterparts.

2) Two-parameter nonlinear scaling model

❖ Simulations of extensive air showers using current hadronic interaction models **predict too small a number of muons compared to observations** by the air-shower experiments, which is known as **the muon deficit problem.** We mimic this effect by construction of mock dataset.



Average logarithm of the muon signal for **Epos-LHC** [8] and **QGSJetII-04** [9]. Solid lines are fits of the function $\bar{S}_{\mu k}$ = const $A^{1-\beta}$ to the TD simulation. From the fit, we obtain **β = 0.925 ± 0.003 for Epos-LHC (red line)**, and **β = 0.918 ± 0.003** for QGSJetII-04 **(blue line)**

$$N_\mu^k = \bar{N}_\mu^k \, A_k^{1-\beta} \, e^{\epsilon_p} \, A_k^{-\Delta\beta}$$

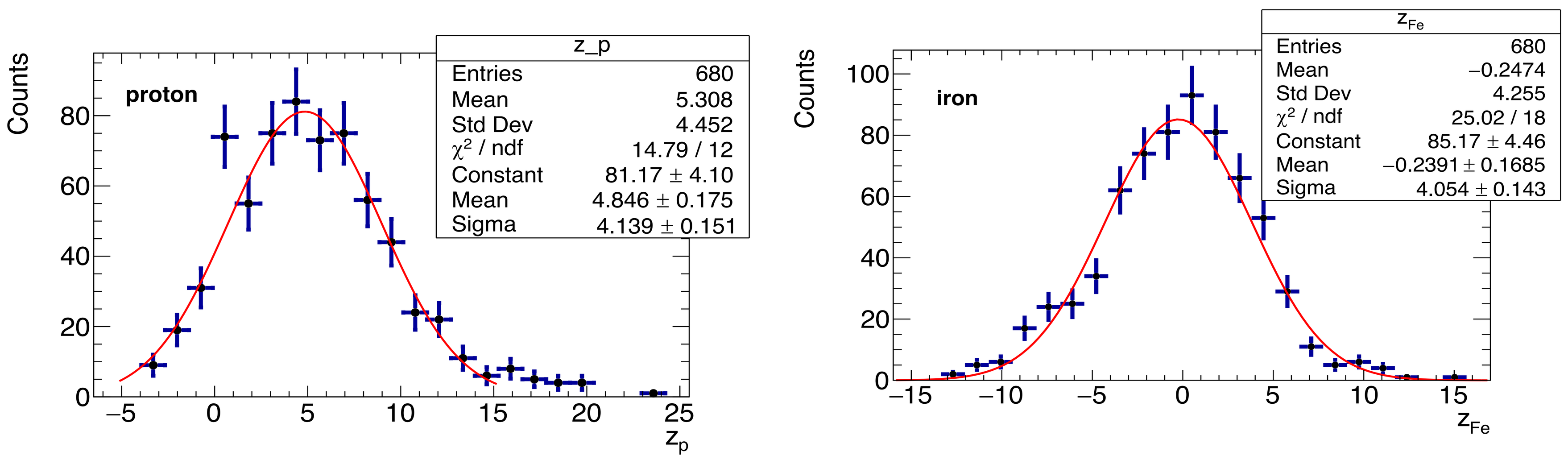
primary mass, index k={p. He, N, Fe}

❖ These two parameters indicate how much we need to scale the proton signal (**ϵ_p term**) and by how much to modify the **β-exponent ($\Delta\beta$)** in the **Heitler-Matthews formula [5]** in order to match the observed numbers of muons in data and in simulations.

3) Individual z_k distributions

$$\mathbf{z_k} := \mathbf{S} - \bar{\mathbf{S}}$$

where S=total signal in the input dataset, Sbar_k= total signal in the matched dataset for primary k



The z_k -distributions for stations at 1000 m from the shower core, from TD simulations at Energy 10^{19} eV for proton (left) and iron (right) induced air showers simulated with **Epos-LHC** and **QGSJetII-04** for mock dataset, see Ref. **[4]** for more details.

❖ **The mean signal $\langle S \rangle$ of the input/matched $\langle \bar{S} \rangle$ dataset is the sum of the mean electromagnetic (em) and muonic components**

$$\langle \mathbf{S} \rangle = \langle \mathbf{S}^{\mathrm{em}} \rangle + \langle \mathbf{S}^{\mu} \rangle \qquad \langle \bar{\mathbf{S}} \rangle = \langle \bar{\mathbf{S}}^{\mathrm{em}} \rangle + \langle \bar{\mathbf{S}}^{\mu} \rangle$$

❖ **Since we assume a perfect matching of the longitudinal profile and thus the EM component of the signal, all the $\langle \bar{S}^{\mathrm{em}} \rangle$ are very close or identical to the $\langle S^{\mathrm{em}} \rangle$ signals in the corresponding input events**

$$\mathbf{\Delta S} := \langle \mathbf{S} \rangle - \langle \bar{\mathbf{S}} \rangle = \sum_{\mathbf{k}} \mathbf{f_k} \left(\langle \mathbf{S}^{\mu}_{\mathbf{k}} \rangle - \langle \bar{\mathbf{S}}^{\mu}_{\mathbf{k}} \rangle \right) = \langle \mathbf{S}^{\mu} \rangle - \langle \bar{\mathbf{S}}^{\mu} \rangle = \mathbf{\Delta S}^{\mu}$$

4) z_{ni}-distribution

❖ Input dataset is constructed from **Epos-LHC [8]** simulations (mock dataset) and is built by taking MC simulations from the TD simulation chain obtained with **Epos-LHC** around 10^{18} eV. The matched dataset is a dataset from **QGSJetII-04 [9]** simulations. The input dataset contains N events and the events will be indexed as $n = 1, \ldots, N$. The multiple longitudinal profile-matched MC events, simulated with primary k, corresponding to an input event n are indexed with $i = 1, \ldots, M$ and are thus denoted with the triplet subscript nki.

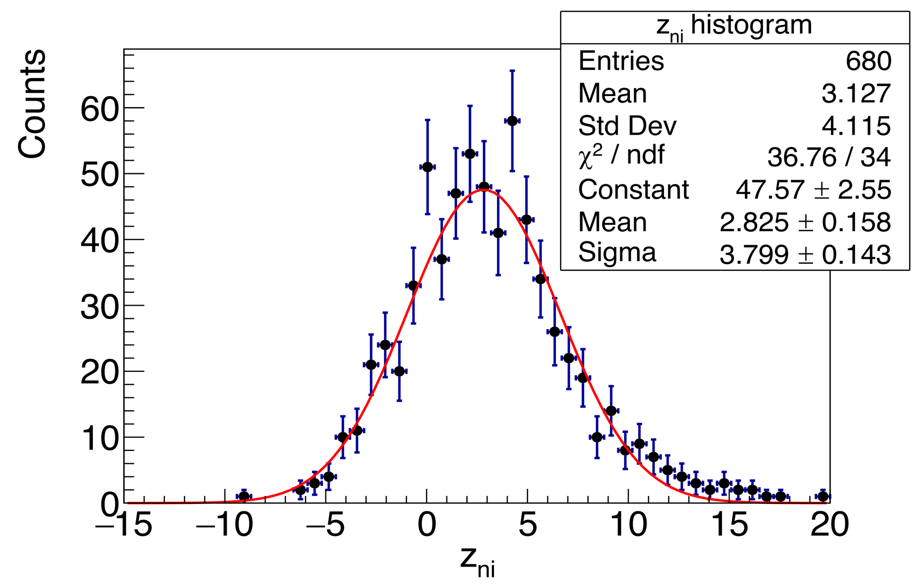
We can define the measured observable:

The signal at 1000 m for the input dataset

The signal at 1000 m for the matched dataset

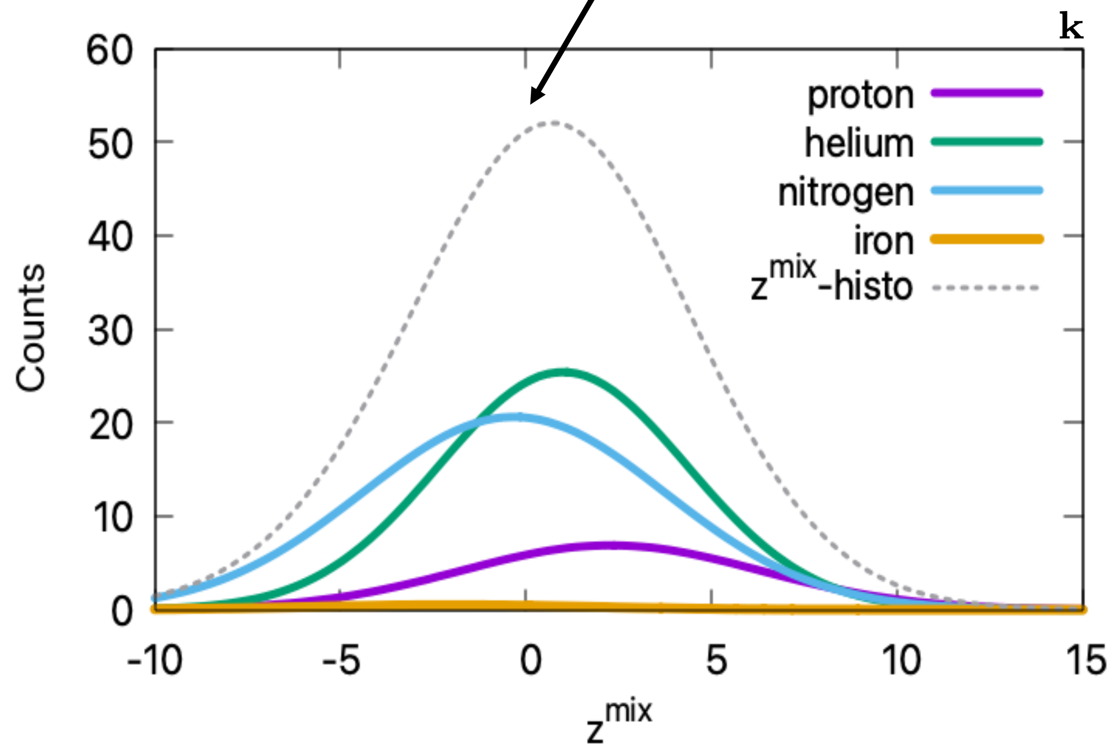
$$\mathbf{z_{ni}} = \mathbf{S_n} - \sum_{\mathbf{k}} \mathbf{f_k} \, \bar{\mathbf{S}}_{\mathbf{nki}}$$

The composition fractions as measured by the Pierre Auger Observatoty at this energy [11]



5) Method: fitting z_{ni}-histogram

$$\langle \mathbf{z_{ni}} \rangle = \langle \mathbf{\Delta S}^{\mu} \rangle = \sum_{\mathbf{k}} \mathbf{f_k} \, \epsilon_{\mathbf{k}} \, \langle \bar{\mathbf{S}}^{\mu}_{\mathbf{k}} \rangle$$



The muon signal at 1000 m for the matched dataset [10]

$$\langle \mathbf{S}^{\mu}_{\mathbf{k}} \rangle = (\mathbf{1} + \epsilon_{\mathbf{k}}) \, \langle \bar{\mathbf{S}}^{\mu}_{\mathbf{k}} \rangle$$

$$\mathbf{1} + \epsilon_{\mathbf{k}} := \mathbf{e}^{\epsilon_{\mathrm{p}}} \, \mathbf{A_k}^{-\mathbf{\Delta}\beta} = \exp(\epsilon_{\mathrm{p}} - \mathbf{\Delta}\beta \ln \mathbf{A_k})$$

$$F(\vec{\varepsilon}, A_{\mathrm{mpl}}) = A_{\mathrm{mpl}} \sum_k f_k \frac{1}{\sqrt{2\pi}\sigma(z_k)} \exp\left[-\frac{(z_{nik} - \varepsilon_k \langle \bar{S}^{\mu} \rangle_k)^2}{2\sigma^2(z_k)} \right], \quad \textbf{Eq. 12}$$

❖ Above formula tells us by how much each z_k -distribution must be shifted, rescaled, and then weighted and summed, in order to retrieve the z_{ni} -distribution and also its first and second moments

6) Results

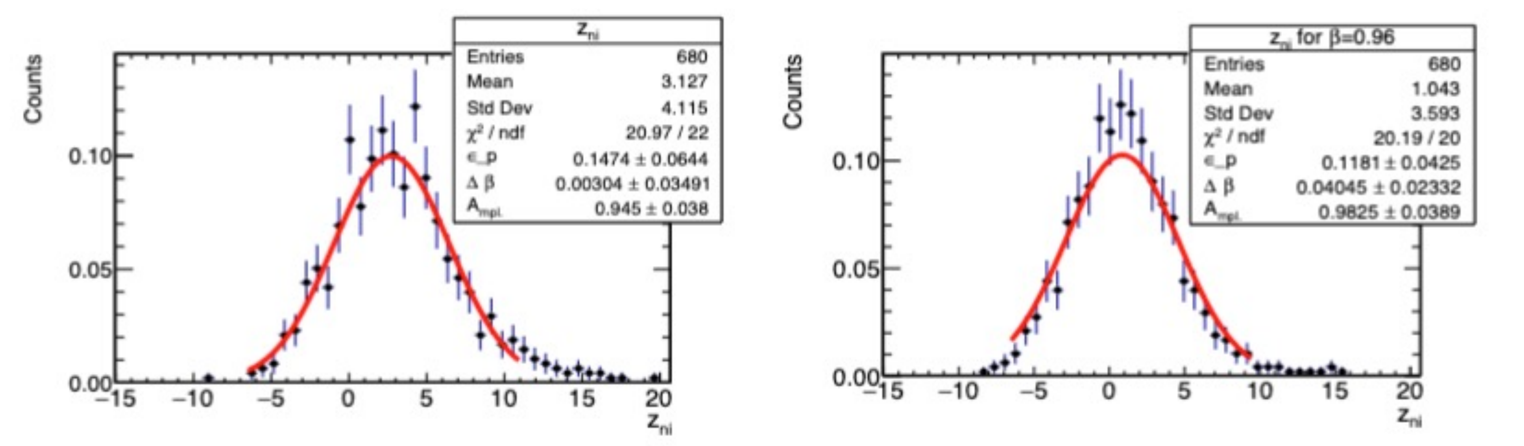


Figure 4: (Left): The results of fitting the function given by Eq. (12) to the normalized z_{ni} histogram shown in Fig. 3 (mockdataset) with $\sigma(z_k)$/VEM = {4.11, 3.40, 4.02, 3.91} and f_k = {0.15, 0.38, 0.46, 01} and $\langle \bar{S}^{\mu} \rangle_k$/VEM = {15.57, 17.25, 19.37, 21.61} and with the two-parameters exponential scaling model. The four individual z_k-distributions match the z_{ni}-histogram for $\varepsilon_p = 0.147 \pm 0.062$, $\Delta\beta = 0.003 \pm 0.035$ and scaling parameter $A_{\mathrm{mpl}} = 0.945 \pm 0.038$. (Right): The four individual z_k-distributions match z_{ni}-histogram for mockdataset with $\beta = 0.96$ and for $\varepsilon_p = 0.118 \pm 0.043$, $\Delta\beta = 0.040 \pm 0.023$ and $A_{\mathrm{mpl}} = 0.983 \pm 0.039$.

k	$r_{\mu,k}$	$\langle \bar{S}^{\mu}_{k} \rangle$/VEM	$\langle S^{\mathrm{rec}}_{\mu,k} \rangle$/VEM	δ	$r_{\mu,k}$	$\langle S^{\mathrm{rec}}_{\mu,k} \rangle$/VEM	δ
p	1.16 ± 0.06	15.57 ± 0.17	18.03 ± 0.18	4.2%	1.13 ± 0.04	17.52 ± 0.17	1.0%
He	1.15 ± 0.06	17.25 ± 0.19	19.90 ± 0.20	4.3%	1.07 ± 0.07	18.11 ± 0.30	1.4%
N	1.15 ± 0.06	19.37 ± 0.20	22.26 ± 0.21	5.3%	1.01 ± 0.09	19.24 ± 0.38	1.9%
Fe	1.14 ± 0.06	21.61 ± 0.23	24.73 ± 0.24	5.6%	0.96 ± 0.10	20.23 ± 0.25	2.4%

Table 1: Values of the muon rescaling factors obtained with the fitting procedure, and of the MC muon signal, the reconstructed muon signals, for all primaries considered and with $f_p = 0.15$, $f_{He} = 0.38$, $f_N = 0.46$, and $f_{Fe} = 0.01$. The overestimation $\delta = (\langle S^{\mathrm{rec}}_{\mu,i} \rangle - \langle S^{\mathrm{mock}}_{\mu,i} \rangle)/\langle S^{\mathrm{mock}}_{\mu,i} \rangle$ of the reconstructed muon signal compared to the one from the mock dataset is also provided. The errors shown in the four column are the square root of the sum of the squares of the errors $\delta r_{\mu,k}$ and $\delta \langle \bar{S}^{\mathrm{rec}}_{\mu,k} \rangle$, i.e. those listed in the second and third columns, respectively. The last free columns show results for mockdataset with $\beta = 0.96$.

7) Summary

❖ The method presented in this work recovers the mean muon signal and provides the ability to calculate muon signals for each element in the considered sample of real-like events.

❖ In this work,we performed calculations of muon scaling factors and β exponents, by fitting a four-element Gaussian distribution to the overall z-histogram, with two-parameter scaling model which should follow Heitler-Matthews progression.

❖ This work shows that the z-method can be applied to hybrid events to determine the muon signal, the scaling factor (total and for each element), and the β exponent.

Acknowledgments: *The authors are very grateful to the Pierre Auger Collaboration for providing the tools necessary for simulation for this contribution. The authors would like to thank the colleagues from the Pierre Auger Collaboration for all the fruitful discussions. We want to acknowledge the support in Poland from the National Science Centre grant* ***No. 2016/23/B/ST9/01635****, grant No.* ***2020/39/B/ST9/01398****, grant No.* ***2022/45/B/ST9/02163*** *and from the Ministry of Education and Science grant No.* ***DIR/WK/2018/11*** *and grant No.* ***2022/WK/12.***

8) References

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